Working memory is the part of memory that has to do with what you are immediately and consciously processing in your mind. It used to be thought that our working memory could hold around seven items, or "chunks," but it's now widely believed that the working memory holds only about four chunks of information. (We tend to automatically group memory items into chunks, so it seems our working memory is bigger than it actually is.[20])

You can think of working memory as the mental equivalent of a juggler. The four items only stay in the air—or in working memory—because you keep adding a little energy. This energy is needed so your metabolic vampires—natural dissipating processes—don't suck the memories away. In other words, you need to maintain these memories actively; otherwise, your body will divert your energy elsewhere, and you'll forget the information you've taken in.

Generally, you can hold about four items in your working memory, as shown in the four-item memory on the left. When you master a technique or concept in math or science, it occupies less space in your working memory. This frees your mental thinking space so that it can more easily grapple with other ideas, as shown on the right.

Your working memory is important in learning math and science because it's like your own private mental blackboard where you can jot down a few ideas that you are considering or trying to understand.

How do you keep things in working memory? Often it's through rehearsal; for example, you can repeat a phone number to yourself until you have a chance to write it down. You may find yourself shutting your eyes to keep any other items from intruding into the limited slots of your working memory as you concentrate.

In contrast, **long-term memory might be thought of as a storage warehouse.** Once items are in there, they generally stay put. The warehouse is large, with room for billions of items, and it can be easy for stored parcels to get buried so deeply that it's difficult to retrieve them. Research has shown that when your brain first puts an item of information in long-term memory, you need to revisit it a few times to increase the chances you'll later be able to find it when you need it.[21] (Techie types sometimes equate short-term memory to random-access memory [RAM], and long-term memory to hard drive space.)

Long-term memory is important for learning math and science because it is where you store the fundamental concepts and techniques that you need to use in problem solving. It takes time to move information from working memory to long-term memory. To help with this process, use a technique called *spaced repetition.* As you may have guessed, this technique involves repeating what you are trying to retain, like a new vocabulary word or a new problem-solving technique, but spacing this repetition out over a number of days.

Putting a day between bouts of repetition—extending your practice over a number of days—does make a difference. Research has shown that if you try to glue things into your memory by repeating something twenty times in one evening, for example, it won't stick nearly as well as it will if you practice it the same number of times over several days or weeks.[22] This is similar to building the brick wall we saw earlier. If you don't leave time for the mortar to dry (time for the synaptic connections to form and strengthen), you won't have a very good structure.

NOW YOU TRY!

Let Your Mind Work in the Background

The next time you are tackling a tough problem, work on it for a few minutes. When you get stuck, move on to another problem. Your diffuse mode can continue working on the tougher problem in the background. When you later return to the tougher problem, you will often be pleasantly surprised by the progress you've made.

ADVICE ON SLEEPING

"Many people will tell you that they can't nap. The one thing I learned from a single yoga class I took many years ago was to slow down my breathing. I just keep breathing slowly in and out and don't think *I must fall asleep*. Instead, I think things like, *Sleepytime!* and just focus on my breathing. I also make sure it's dark in the room, or I cover my eyes with one of those airplane sleep masks. Also, I set my phone alarm for twenty-one minutes because turning a short power nap into a longer sleep can leave you groggy. This amount of time gives me what's basically a cognitive reboot."

—*Amy Alkon, syndicated columnist and catnap queen*

The Importance of Sleep in Learning

You may be surprised to learn that simply being awake creates toxic products in your brain. During sleep, your cells shrink, causing a striking increase in the space *between* your cells. This is equivalent to turning on a faucet—it allows fluid to wash past and push the toxins

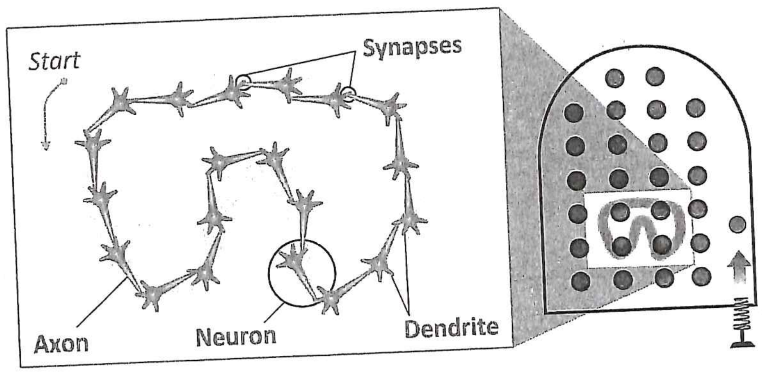
out.[23] This nightly housecleaning is part of what keeps your brain healthy. When you get too little sleep, the buildup of these toxic products is believed to explain why you can't think very clearly. (Too little sleep is affiliated with conditions ranging from Alzheimer's to depression—prolonged sleeplessness is lethal.)

Studies have shown that sleep is a vital part of memory and learning.[24] Part of what this special sleep-time tidying does is erase trivial aspects of memories and simultaneously strengthen areas of importance. During sleep, your brain also rehearses some of the tougher parts of whatever you are trying to learn—going over and over neural patterns to deepen and strengthen them.[25]

Finally, sleep has been shown to make a remarkable difference in people's ability to figure out difficult problems and to find meaning and understanding in what they are learning. It's as if the complete deactivation of the conscious "you" in the prefrontal cortex helps other areas of the brain start talking more easily to one another, allowing them to put together the neural solution to your problem as you sleep.[26] (Of course, you must plant the seed for your diffuse mode by first doing focused-mode work.) It seems that if you go over the material right before taking a nap or going to sleep for the evening, you have an increased chance of dreaming about it. If you go even further and set it in mind that you *want* to dream about the material, it seems to improve your chances of dreaming about it still further.[27] Dreaming about what you are studying can substantially enhance your ability to understand—it somehow consolidates your memories into easier-to-grasp chunks.[28]

If you're tired, it's often best to just go to sleep and get up a little earlier the next day, so your reading is done with a better-rested brain. Experienced learners will attest to the fact that reading for one hour with a well-rested brain is better than reading for three hours with a tired brain. A sleep-deprived brain simply can't make connections.

~~and kicks become incorporated into larger, more creative interpretations.~~



The left image symbolizes the compact connections when one chunk of knowledge is formed—neurons that fire together wire together. The image on the right shows the same pattern in your mind's symbolic pinball machine. Such a memory trace is easy to recall when you need it.

What Is a Chunk? Solomon's Chunking Problem

Solomon Shereshevsky's extraordinary memory came with a surprising drawback. His individual memory traces were each so colorful and emotional—so rich with connections—that they interfered with his ability to put those traces together and create conceptual **chunks**. He couldn't see the forest, in other words, because his imagery of each of the individual trees was so vivid.

Chunks are pieces of information that are bound together through meaning. You can take the letters *p*, *o*, and *p* and bind them into one conceptual, easy-to-remember chunk, the word *pop*. It's like converting a cumbersome computer file into a .zip file. Underneath that simple *pop* chunk is a symphony of neurons that have learned to trill in tune with one another. The complex neural activity that ties together our simplifying, abstract chunks of thought—

whether those thoughts pertain to acronyms, ideas, or concepts—are the basis of much of science, literature, and art.

Let's take an example. In the early 1900s, German researcher Alfred Wegener put together his theory of continental drift. As Wegener analyzed maps and thought about the information he'd gleaned from his studies and exploration, he realized that the different land masses fit together like puzzle pieces. The similarity of rocks and fossils between the land masses reinforced the fit. Once Wegener put the clues together, it was clear that all the continents had once, very long ago, been joined together in a single landmass. Over time, the mass had broken up and the pieces had drifted apart to form the continents separated by oceans we see today.

Continental drift! Wow—what a great discovery!

But if Solomon Shereshevsky had read this same story about the discovery of continental drift, he wouldn't have gotten the point. Even though he would have been able to repeat every individual word in the story, the concept of continental drift would have been very difficult for him to grasp, since he was unable to link his individual memory traces together to create conceptual chunks.

As it turns out, **one of the first steps toward gaining expertise in math and science is to create conceptual chunks—mental leaps that unite separate bits of information through meaning.**[4] Chunking the information you deal with helps your brain run more efficiently. Once you chunk an idea or concept, you don't need to remember all the little underlying details; you've got the main idea—the chunk—and that's enough. It's like getting dressed in the morning. Usually you just think one simple thought—*I'll get dressed.* But it's amazing when you realize the complex swirl of underlying activities that take place with that one simple chunk of a thought.

When you are studying math and science, then, how do you form a chunk?

Basic Steps to Forming a Chunk

Chunks related to different concepts and procedures can be molded in many different ways. It's often quite easy. You formed a simple chunk, for example, when you grasped the idea of continental drift. But since this is a book about how to learn math and science in general rather than geology in particular, we're going to take as our initial, illustrative chunk *the ability to understand and work a certain type of math or science problem.*

When you are learning new math and science material, you are almost always given sample problems with worked-out solutions. This is because, when you are first trying to understand how to work a problem, you have a heavy cognitive load—so it helps to start out with a fully worked-through example. It's like using a GPS unit when you are driving on unfamiliar roads in the middle of the night. Most of the details in the worked-out solution are right there, and your task is simply to figure out why the steps are taken the way they are. That can help you see the key features and underlying principles of a problem.

Some instructors do not like to give students extra worked-out problems or old tests, as they think it makes matters too easy. But there is bountiful evidence that having these kinds of resources available helps students learn much more deeply.[5] The one concern about using worked-out examples to form chunks is that it can be all too easy to focus too much on why an individual step works and not on the *connection* between steps—that is, on why this particular step is the next thing you should do. So keep in mind that I'm not talking about a cookie-cutter "just do as you're told" mindless approach when following a worked-out solution. It's more like using a guide to help you when traveling to a new place. Pay attention to

When you first look at a brand-new concept in science or math, it sometimes doesn't make much sense, as shown by the puzzle pieces above on the left. Just memorizing a fact *(center)* without understanding or context doesn't help you understand what's really going on, or how the concept fits together with the other concepts you are learning—notice there are no interlocking puzzle edges on the piece to help you fit into other pieces. **Chunking** *(right)* is the mental leap that helps you unite bits of information together through meaning. The new logical whole makes the chunk easier to remember, and also makes it easier to fit the chunk into the larger picture of what you are learning.

what's going on around you when you're with the guide, and soon you'll find yourself able to get there on your own. You will even begin to figure out new ways of getting there that the guide didn't show you.

1. **The first step in chunking, then, is to simply *focus your attention* on the information you want to chunk.**[6] If you have the television going in the background, or you're looking up every few minutes to check or answer your phone or computer messages, it means that you're going to have difficulty making a chunk, because your brain is not really focusing on the chunking. When you first begin to learn something, you are making new neural patterns and connecting them with preexisting patterns that are spread through many areas of the brain.[7] Your octopus tentacles can't make connections very well if some of them are off on other thoughts.

2. **The second step in chunking is to *understand* the basic idea you are trying to chunk,** whether it is understanding a concept such as continental drift, the idea that force is proportional to mass, the economic principle of supply and demand, or a particular type of math problem. Although this step of basic understanding—synthesizing the gist of what's important—was difficult for Solomon Shereshevsky, most students figure out these main ideas naturally. Or at least, they can grasp those ideas if they allow the focused and diffuse modes of thinking to take turns in helping them figure out what's going on.

Understanding is like a superglue that helps hold the underlying memory traces together. It creates broad, encompassing traces that link to many memory traces.[8] Can you create a chunk if you don't understand? Yes, but it's a useless chunk that won't fit in with other material you are learning.

That said, it's important to realize that *just understanding how a problem was solved does not necessarily create a chunk that you can easily call to mind later.* Do *not* confuse the "aha!" of a breakthrough in understanding with solid expertise! (That's part of why you can grasp an idea when a teacher presents it in class, but if you don't review it fairly soon after you've first learned it, it can seem incomprehensible when it comes time to prepare for a test.) Closing the book and testing yourself on how to solve the problems will also speed up your learning at this stage.

3. **The third step to chunking is gaining context so you see not just how, but also *when* to use this chunk.** Context means going beyond the initial problem and seeing more

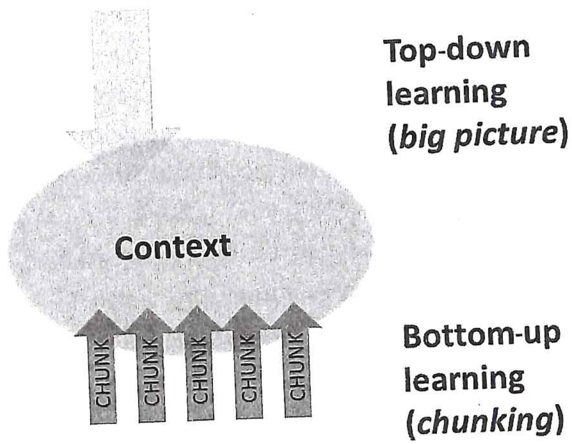
broadly, repeating and practicing with both related and unrelated problems so you see not only when to use the chunk, but when *not* to use it. This helps you see how your newly formed chunk fits into the bigger picture. In other words, you may have a tool in your strategy or problem-solving toolbox, but if you don't know when to use that tool, it's not going to do you a lot of good. Ultimately, practice helps you broaden the networks of neurons connected to your chunk, ensuring that it is not only firm, but also accessible from many different paths.

There are chunks related to both concepts and procedures that reinforce one another. Solving a lot of math problems provides an opportunity to learn why the procedure works the way it does or why it works at all. Understanding the underlying concept makes it easier to detect errors when you make them. (Trust me, you *will* make errors, and that's a good thing.) It also makes it much easier to apply your knowledge to novel problems, a phenomenon called *transfer.* We'll talk more about transfer later.

As you can see from the following "top-down, bottom-up" illustration, learning takes place in two ways. There is a **bottom-up chunking process** where practice and repetition can help you both build and strengthen each chunk, so you can easily gain access to it when needed. And there is a **top-down "big picture" process** that allows you to see where what you are learning fits in.[9] *Both processes are vital in gaining mastery over the material.* Context is where bottom-up and top-down learning meet. To clarify here—chunking may involve your learning *how* to use a certain problem-solving technique. Context means learning *when* to use that technique instead of some other technique.

Those are the essential steps to making a chunk and fitting that chunk into a greater conceptual overview of what you are learning.

But there's more.

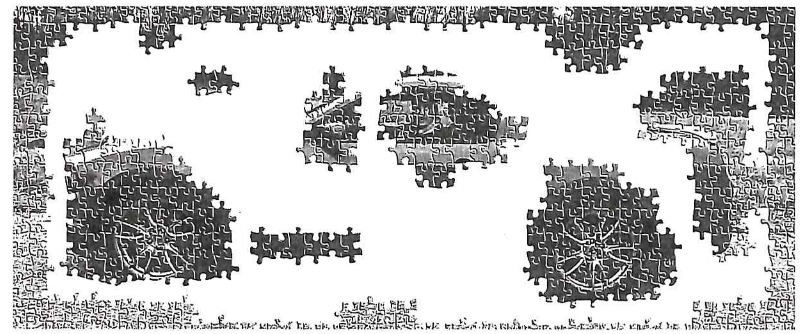


Both top-down, big-picture learning, and bottom-up chunking are important in becoming an expert in math and science.

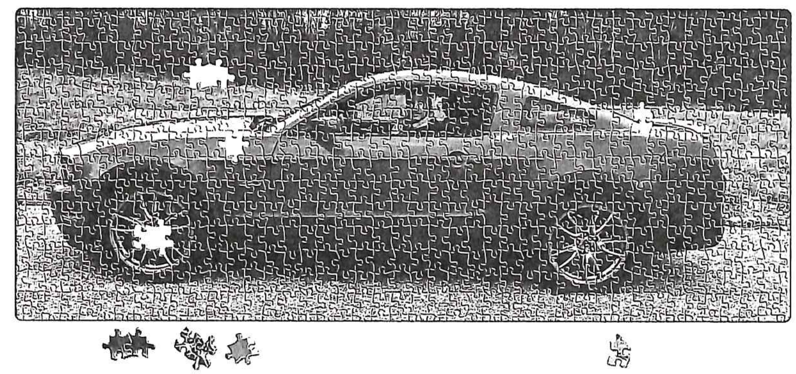
NOW I LAY ME DOWN TO SLEEP

"I tell my students that internalizing the accounting fundamentals is like internalizing how to type on a keyboard. In fact, as I write this myself, I'm not thinking of the act of typing, but of formulating my thoughts—the typing comes naturally. My mantra at the end of each class is to tell students to look at the Debit and Credit Rules as well as the Accounting Equation just before they tuck themselves in at night. Let those be the last things they repeat to themselves before falling asleep. Well, except meditation or prayers, of course!"

—*Debra Gassner Dragone, Accounting Instructor, University of Delaware*



Skimming through a chapter or listening to a very well-organized lecture can allow you to gain a sense of the big picture. This can help you know where to put the chunks you are constructing. Learn the major concepts or points first—these are often the key parts of a good instructor or book chapter's outline, flow charts, tables, or concept maps. Once you have this done, fill in the details. Even if a few of the puzzle pieces are missing at the end of your studies, you can still see the big picture.



Illusions of Competence and the Importance of Recall

Attempting to *recall* the material you are trying to learn—retrieval practice—is far more effective than simply rereading the material.[10] Psychologist Jeffrey Karpicke and his colleagues have shown that many students experience *illusions of competence* when they are studying. Most students, Karpicke found, "repeatedly read their notes or textbook (despite the limited benefits of this strategy), but relatively few engage in self-testing or retrieval practice while studying."[11] When you have the book (or Google!) open right in front of you, it provides the illusion that the material is also in your brain. *But it's not.* Because it can be easier to look at the book instead of recalling, students persist in their illusion—studying in a far less productive way.

This, indeed, is why just *wanting* to learn the material, and spending a lot of time with it, doesn't guarantee you'll actually learn it. As Alan Baddeley, a renowned psychologist and expert on memory, notes: **"Intention to learn is helpful only if it leads to the use of good learning strategies."**[12]

You may be surprised to learn that highlighting and underlining must be done carefully—otherwise they can be not only ineffective but also misleading. It's as if the motion of your hand can fool you into thinking you've placed the concept in your brain. When marking up the text, train yourself to look for main ideas before making any marks, and keep your text markings to a minimum—one sentence or less per paragraph.[13] Words or notes in a margin that synthesize key concepts are a good idea.[14]

Using recall—mental retrieval of the key ideas—rather than passive rereading will make your study time more focused and effective. The only time rereading text seems to be effective is if you let time pass between rereadings so that it becomes more of an exercise in spaced repetition.[15]

Along these same lines, always work through homework problems in math and science on your own. Some textbooks include solutions at the back of the book, but you should look at these only to check your answer. This will help ensure that the material is more deeply rooted in your mind and make it much more accessible when you really need it. This is why instructors place so much emphasis

on showing your work and giving your reasoning on tests and homework problems. Doing so forces you to think your way through a problem and provides a self-test of your understanding. This additional information about your thinking also gives graders a better opportunity to provide useful feedback.

You don't want to wait too long for the recall practice, so that you have to start the reinforcement of the concept from scratch every time. Try to touch again on something you're learning within a day, especially if it's new and rather challenging. This is why many professors recommend that, if at all possible, you rewrite your notes during the evening after a lecture. This helps to solidify newly forming chunks and reveals the holes in your understanding that professors just *love* to target on tests. Knowing where the holes are, of course, is the first step toward getting them filled in.

Once you've got something down, you can expand the time between "upkeep" repetitions to weeks or months—and eventually it can become close to permanent. (Returning to Russia on a visit, for example, I found myself annoyed by an unscrupulous taxi driver. To my amazement, words I hadn't thought or used for twenty-five years popped from my mouth—I hadn't even been consciously aware I *knew* those words!)

MAKE YOUR KNOWLEDGE SECOND NATURE

"Getting a concept in class versus being able to apply it to a genuine physical problem is the difference between a simple student and a full-blown scientist or engineer. The only way I know of to make that jump is to work with the concept until it becomes second nature, so you can begin to use it like a tool."

—Thomas Day, Professor of Audio Engineering, McNally Smith College of Music

~~problem solving for you and will soon find that different solution techniques are lurking at the edge of your memory. Before midterms or finals, it is easy to brush up and have these solutions at the mental ready.~~

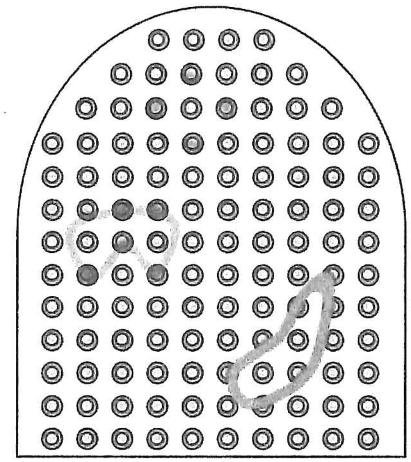
NOW YOU TRY!

What to Do If You Can't Grasp It

If you don't understand a method presented in a course you are taking, stop and work backward. Go to the Internet and discover who first figured out the method or some of the earliest people to use it. Try to understand how the creative inventor arrived at the idea and why the idea is used—you can often find a simple explanation that gives a basic sense of why a method is being taught and why you would want to use it.

Practice Makes Permanent

I've already mentioned that just *understanding* what's going on is *not* usually enough to create a chunk. You can get a sense of what I mean by looking at the "brain" picture shown on p. 69. The chunks (loops) shown are really just extended memory traces that have arisen because you have knit together an understanding. A chunk, in other words, is simply a more complex memory trace. At the top is a faint chunk. That chunk is what begins to form after you've understood a concept or problem and practiced just a time or two. In the middle, the pattern is darker. This is the stronger neural pattern that results after you've practiced a little more and seen the chunk in more contexts. At the bottom, the chunk is very dark.



Solving problems in math and science is like playing a piece on the piano. The more you practice, the firmer, darker, and stronger your mental patterns become.

You've now got a solid chunk that's firmly embedded in long-term memory.

Incidentally, strengthening an initial learning pattern within a day after you first begin forming it is important. Without the strengthening, the pattern can quickly fade away. Later, we'll talk more about the importance of spaced repetition in learning. Also, you can reinforce a "wrong" process by doing the same problems over and over the wrong way. This is why checking things is so important. Even getting the right answer can occasionally mislead you if you get it by using an incorrect procedure.

THE IMPORTANCE OF CHUNKING

"Mathematics is amazingly compressible: you may struggle a long time, step by step, to work through the same process or idea from several approaches. But once you really understand it and have the mental perspective to see it as a whole, there is often a tremendous mental compression. You can file it away, recall it quickly and completely when you need it, and use it as just one step in some other mental process. The insight that goes with this compression is one of the real joys of mathematics."[26]

—*William Thurston, winner of the Fields Medal, the top award in mathematics*

The challenge with repetition and practice, which lie behind the mind's creation of solid chunks, is that it can be boring. Worse yet, in the hands of a poor instructor, like my old math teacher, Mr. Crotchety, practice can become an unrelenting instrument of torture. Despite its occasional misuse, however, it's critical. Everybody knows you can't effectively learn the chunked patterns of chess, language, music, dance—just about anything worthwhile—without repetition. Good instructors can explain why the practice and repetition is worth the trouble.

Ultimately, both bottom-up chunking and top-down big-picture approaches are vital if you are to become an expert with the material. We love creativity and the idea of being able to learn by seeing the big picture. **But you can't learn mathematics or science without also including a healthy dose of practice and repetition to help you build the chunks that will underpin your expertise.**[27]

Research published in the journal *Science* provided solid evidence along these lines.[28] Students studied a scientific text and then

practiced it by recalling as much of the information as they could. Then they restudied the text and recalled it (that is, tried to remember the key ideas) once more.

The results?

In the same amount of time, *by simply practicing and recalling the material,* students learned far more and at a much deeper level than they did using any other approach, including simply rereading the text a number of times or drawing concept maps that supposedly enriched the relationships in the materials under study. This improved learning comes whether students take a formal test or just informally test themselves.

This reinforces an idea we've alluded to already. When we retrieve knowledge, we're not being mindless robots—*the retrieval process itself enhances deep learning and helps us begin forming chunks.*[29] Even more of a surprise to researchers was that the students themselves predicted that simply reading and recalling the materials wasn't the best way to learn. They thought concept mapping (drawing diagrams that show the relationship between concepts) would be best. But if you try to build connections between chunks *before the basic chunks are embedded in the brain,* it doesn't work as well. It's like trying to learn advanced strategy in chess before you even understand the basic concepts of how the pieces move.[30]

Practicing math and science problems and concepts in a variety of situations helps you build chunks—solid neural patterns with deep, contextual richness.[31] The fact is, when learning *any* new skill or discipline, you need plenty of varied practice with different contexts. This helps build the neural patterns you need to make the new skill a comfortable part of your way of thinking.

KEEP YOUR LEARNING AT THE TIP OF YOUR TONGUE

"By chance, I have used many of the learning techniques described in this book. As an undergraduate I took physical chemistry and became fascinated with the derivations. I got into a habit of doing every problem in the book. As a result, I hardwired my brain to solve problems. By the end of the semester I could look at a problem and know almost immediately how to solve it. I suggest this strategy to my science majors in particular, but also to the nonscientists. I also talk about the need to study every day, not necessarily for long periods of time but just enough to keep what you are learning at the tip of your tongue. I use the example of being bilingual. When I go to France to work, my French takes a few days to kick in, but then it is fine. When I return to the States and a student or colleague asks me something on my first or second day back, I have to search for the English words! When you practice every day the information is just there—you do not have to search for it."

—Robert R. Gamache, Associate Vice President, Academic Affairs, Student Affairs, and International Relations, University of Massachusetts, Lowell

Recall Material While Outside Your Usual Place of Study: The Value of Walking

Doing something physically active is especially helpful when you have trouble grasping a key idea. As mentioned earlier, stories abound of innovative scientific breakthroughs that occurred when the people who made them were out walking.[32]

In addition, **recalling material when you are outside your usual place of study helps you strengthen your grasp of the material by viewing it from a different perspective.** People sometimes lose subconscious cues when they take a test in a room that looks different

from where they studied. By thinking about the material while you are in various physical environments, you become independent of cues from any one location, which helps you avoid the problem of the test room being different from where you originally learned the material.[33]

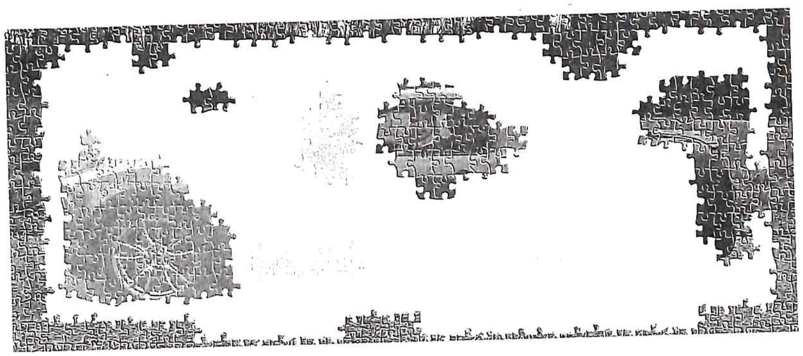
Internalizing math and science concepts can be *easier* than memorizing a list of Chinese vocabulary words or guitar chords. After all, you've got the problem there to speak to you, telling you what you need to do next. In that sense, problem solving in math and science is like dance. In dance, you can *feel* your body hinting at the next move.

Different types of problems have different review time frames that are specific to your own learning speed and style.[34] And of course, you have other obligations in your life besides learning one particular topic. You have to prioritize how much you're able to do, also keeping in mind that you *must* schedule some time off to keep your diffuse mode in play. How much internalizing can you do at a stretch? It depends—everyone is different. But, here's the real beauty of internalizing problem solutions in math and science. The more you do it, the easier it becomes, and the more useful it is.

ORGANIZE, CHUNK—AND SUCCEED

"The first thing I always do with students who are struggling is ask to see how they are organizing their notes from class and reading. We often spend most of the first meeting going over ways they can organize or chunk their information rather than with my explaining concepts. I have them come back the next week with their material already organized, and they are amazed at how much more they retain."

—Jason Dechant, Ph.D., Course Director, Health Promotion and Development, School of Nursing, University of Pittsburgh



If you don't practice with your growing chunks, it is harder to put together the big picture—the pieces are simply too faint.

Interleaving—Doing a Mixture of Different Kinds of Problems—versus Overlearning

One last important tip in becoming an equation whisperer is interleaving.[35] **Interleaving means practice by doing a mixture of different kinds of problems requiring different strategies.**

When you are learning a new problem-solving approach, either from your teacher or from a book, you tend to learn the new technique and then practice it over and over again during the same study session. Continuing the study or practice after it is well understood is called *overlearning*. Overlearning can have its place—it can help produce an automaticity that is important when you are executing a serve in tennis or playing a perfect piano concerto. But be wary of repetitive overlearning during a single session in math and science learning—research has shown it can be a waste of valuable learning time.[36] (Revisiting the approach mixed with other approaches during a subsequent study session, however, is just fine.)

In summary, then, once you've got the basic idea down during a session, continuing to hammer away at it during the same session doesn't necessarily strengthen the kinds of long-term memory con-

nections you want to have strengthened. Worse yet, focusing on one technique is a little like learning carpentry by only practicing with a hammer. After a while, you think you can fix anything by just bashing it.[37]

The reality is, mastering a new subject means learning to select and use the proper technique for a problem. The only way to learn that is by practicing with problems that require *different* techniques. Once you have the basic idea of a technique down during your study session (sort of like learning to ride a bike with training wheels), start interleaving your practice with problems of different types.[38] Sometimes this can be a little tough to do. A given section in a book, for example, is often devoted to a specific technique, so when you flip to that section, you already know which technique you're going to use.[39] Still, do what you can to mix up your learning. It can help to look ahead at the more varied problem sets that are sometimes found at the end of chapters. Or you can deliberately try to make yourself occasionally pick out why some problems call for one technique as opposed to another. **You want your brain to become used to the idea that just knowing *how* to use a particular problem-solving technique isn't enough—you also need to know *when* to use it.**

Consider creating index cards with the problem question on one side, and the question and solution steps on the other. That way you can easily shuffle the cards and be faced with a random variety of techniques you must call to mind. When you first review the cards, you can sit at a desk or table and see how much of the solution you can write on a blank sheet of paper without peeking at the back of the card. Later, when mastery is more certain, you can review your cards anywhere, even while out for a walk. Use the initial question as a cue to bring to mind the steps of the response, and flip the card over if necessary to verify that you've got the procedural steps all in mind. You are basically strengthening a new chunk. Another idea is to open the book to a randomly chosen page and work a problem while, as much as possible, hiding from view everything but the problem.

EMPHASIZE INTERLEAVING INSTEAD OF OVERLEARNING

Psychologist Doug Rohrer of the University of South Florida has done considerable research on overlearning and interleaving in math and science. He notes:

> "Many people believe overlearning means studying or practicing until mastery is achieved. However, in the research literature, overlearning refers to a learning strategy in which a student continues to study or practice immediately after some criterion has been achieved. An example might be correctly solving a certain kind of math problem and then immediately working several more problems of the same kind. Although working more problems of the same kind (rather than fewer) often boosts scores on a subsequent test, doing too many problems of the same kind in immediate succession provides diminishing returns.
>
> "In the classroom and elsewhere, students should maximize the amount they learn per unit time spent studying or practicing—that is, they should get the most bang for the buck. How can students do this? The scientific literature provides an unequivocal answer: Rather than devote a long session to the study or practice of the same skill or concept so that overlearning occurs, students should divide their effort across several shorter sessions. This doesn't mean that long study sessions are necessarily a bad idea. Long sessions are fine as long as students don't devote too much time to any one skill or concept. Once they understand 'X,' they should move on to something else and return to 'X' on another day."[40]

It's best to write the initial solution, or diagram, or concept, out by hand. There's evidence that writing by hand helps get the ideas into

mind more easily than if you type the answer.[41] More than that, it's often easier to write symbolic material like Σ or Ω by hand than to search out the symbol and type it (unless you use the symbols often enough to memorize the alt codes).[42] But if you then want to photograph or scan the question and your handwritten solution to load it into a flash card program for your smartphone or laptop, that will work just fine. Beware—a common illusion of competence is to continue practicing a technique you know, simply because it's easy and it feels good to successfully solve problems. Interleaving your studies—making a point to review for a test, for example, by skipping around through problems in the different chapters and materials—can sometimes seem to make your learning more difficult. But in reality, it helps you learn more deeply.

AVOID MIMICKING SOLUTIONS—PRACTICE CHANGING MENTAL GEARS

> "When students do homework assignments, they often have ten identical problems in a row. After the second or third problem, they are no longer thinking; they are mimicking what they did on the previous problem. I tell them that, when doing the homework from section 9.4, after doing a few problems, go back and do a problem from section 9.3. Do a couple more 9.4 problems, and then do one from section 9.1. This will give them practice in mentally shifting gears in the same way they'll need to switch gears on the test.
>
> "I also believe too many students do homework just to get it done. They finish a problem, check their answer in the back of the text, smile, and go on to the next problem. I suggest that they insert a step between the smile and going on to the next problem—asking themselves this question: How would I know how to do the problem this way if I saw it on a test mixed together with other problems and I didn't know it was from this

{ 11 } more memory tips

Create a Lively Visual Metaphor or Analogy

One of the best things you can do to not only remember but *understand* concepts in math and science is to **create a metaphor or analogy for it**—often, the more visual, the better.[1] A metaphor is just a way of realizing that one thing is somehow similar to another. Simple ideas like one geography teacher's description of Syria as shaped like a bowl of cereal and Jordan as a Nike Air Jordan sneaker can stick with a student for decades.

If you're trying to understand electrical current, it can help to visualize it as water. Similarly, electrical voltage can "feel like" pressure. Voltage helps push the electrical current to where you want it to go, just like a mechanical pump uses physical pressure to push real water. As you climb to a more sophisticated understanding of electricity, or whatever topic you are concentrating on, you can

revise your metaphors, or toss them away and create more meaningful ones.

If you are trying to understand the concept of limits in calculus, you might visualize a runner heading for the finish line. The closer the runner gets, the slower he goes. It's one of those slo-mo camera shots where the runner is never quite able to reach the ribbon, just as we might not quite be able to get to the actual limit. Incidentally, the little book *Calculus Made Easy*, by Silvanus Thompson, has helped generations of students master the subject. Sometimes textbooks can get so focused on all the details that you lose sight of the most important, big-picture concepts. Little books like *Calculus Made Easy* are good to dip into because they help us focus in a simple way on the most important issues.

It's often helpful to pretend *you* are the concept you are trying to understand. Put yourself in an electron's warm and fuzzy slippers as it burrows through a slab of copper, or sneak inside the *x* of an algebraic equation and feel what it's like to poke your head out of the rabbit hole (just don't let it get exploded with an inadvertent "divide by zero").

MOONBEAMS AND SCHOOL DREAMS

"I always study before I go to bed. For some reason, I usually dream about the material I just studied. Most times these 'school dreams' are quite strange but helpful. For instance, when I was taking an operations research class, I would dream I was running back and forth between nodes, physically acting out the shortest path algorithm. People think I'm crazy, but I think it's great; it means I don't have to study as much as other people do. I guess these dreams involve my subconsciously making metaphors."

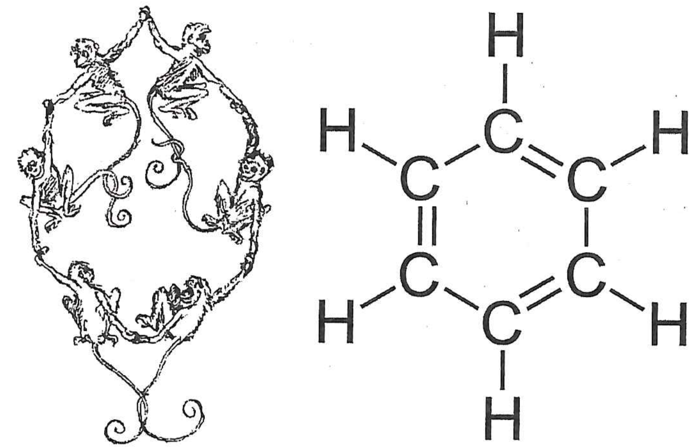
—*Anthony Sciuto, senior, industrial and systems engineering*

In chemistry, compare a cation with a cat that has paws and is therefore "pawsitive," and an anion with an onion that is negative because it makes you cry.

Metaphors are never perfect. But then, *all* scientific models are just metaphors, which means they also break down at some point.[2] But never mind that—metaphors (and models!) are vitally important in giving a physical understanding of the central idea behind the mathematical or scientific process or concept that you are trying to understand. Interestingly, metaphors and analogies are useful for getting people out of *Einstellung*—being blocked by thinking about a problem in the wrong way. For example, telling a simple story of soldiers attacking a fortress from many directions at once can open creative paths for students to intuit how many low-intensity rays can be effectively used to destroy a cancerous tumor.[3]

Metaphors also help glue an idea in your mind, because they make a connection to neural structures that are already there. It's like being able to trace a pattern with tracing paper—metaphors at least help you get a sense of what's going on. If there's a time when you can't think of a metaphor, just put a pen or pencil in your hand and a sheet of paper in front of you. Whether using words or pictures, you will often be amazed at what just noodling about for a minute or two will bring.

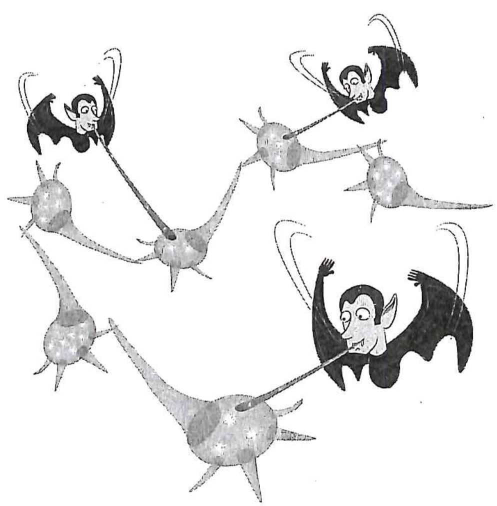
METAPHORS AND VISUALIZATION IN SCIENCE



Metaphors and visualization—being able to see something in your mind's eye—have been uniquely powerful in helping the scientific and engineering world move forward.[4] In the 1800s, for example, when chemists began to imagine and visualize the miniature world of molecules, dramatic progress began to be made. Here is a delightful illustration of monkeys in a benzene ring from an insider spoof of German academic chemical life, printed in 1886.[5] Note the single bonds with the monkeys' hands and the double bonds with their tiny tails.

Spaced Repetition to Help Lodge Ideas in Memory

Focusing your attention brings something into your temporary working memory. But for that "something" to move from working memory to long-term memory, two things should happen: the idea should be **memorable** (*there's a gigantic flying mule braying $f = ma$ on my couch!*), and it must be **repeated.** Otherwise, your natural metabolic processes, like tiny vampires, simply suck away faint, newly forming patterns of connections. This vampiric removal of faint patterns is actually a good thing. Much of what goes on around you is basically trivial—if you remembered it all, you'd end up like a hoarder, trapped in an immense collection of useless memories.



If you don't make a point of repeating what you want to remember, your "metabolic vampires" can suck away the neural pattern related to that memory before it can strengthen and solidify.

Repetition is important; even when you make something memorable, repetition helps get that memorable item firmly lodged in long-term memory. But how many times should you repeat? How long should you wait between repetitions?[6] And is there anything you can do to make the repetition process more effective?

Research has given us helpful insight. Let's take a practical example. Say you want to remember information related to the concept of *density*—namely that it is symbolized by a funny-looking symbol, ρ, which is pronounced "row," and that it is measured in standard units of "kilograms per cubic meter."

How can you conveniently and effectively cement this information into memory? (You know now that placing small chunks of information like this in your long-term memory helps gradually build your big-picture understanding of a subject.)

You might take an index card and write "ρ" on one side and the remaining information on the other. **Writing appears to help you to more deeply encode (that is, convert into neural memory structures) what you are trying to learn.** While you are writing out "kilograms per cubic meter," you might imagine a shadowy kilogram (just feel that mass!) lurking in an oversized piece of luggage that happens to be a meter on each side. The more you can turn what you are trying to remember into something memorable, the easier it will be to recall. You will want to say the word and its meaning aloud, to start setting auditory hooks to the material.

Next, just look at the side of the card with the "ρ" on it and see whether you can remember what's on the other side of the card. If you can't, flip it over and remind yourself of what you are supposed to know. If you can remember, put the card away.

Now do something else—perhaps prepare another card and test yourself on it. Once you have several cards together, try running through them all to see if you can remember them. (This helps you interleave your learning.) Don't be surprised if you struggle a bit.

Once you've given your cards a good try, put them away. Wait and take them out again before you go to sleep. Remember that sleep is when your mind repeats patterns and pieces together solutions.

Briefly repeat what you want to remember over several days; perhaps for a few minutes each morning or each evening, change the order of your cards sometimes. Gradually extend the times between repetitions as the material firms itself into your mind. By increasing your spacing as you become more certain of mastery, you will lock the material more firmly into place.[7] (Great flash card systems like Anki have built in algorithms that repeat on a scale ranging from days to months.)

Interestingly, one of the best ways to remember people's names is to simply try to retrieve the people's names from memory at increasing time intervals after first learning the name.[8] Material that you do not review is more easily discounted or forgotten. Your metabolic vampires suck away the links to the memories. This is why **it's wise to be careful about what you decide to skip when reviewing for tests. Your memory for related but nonreviewed material can become impaired.**[9]

SPACED REPETITION—
USEFUL FOR BOTH STUDENTS AND PROFESSORS!

"I have been advising my students to do spaced repetition over days and weeks, not just in my analytic courses, but also in my History of Ancient Engineering course. When memorizing strange names and terms, it's always best to practice over several days. In fact, that's precisely what I do when I'm preparing for lectures—repeat the terms out loud over a period of several days, so they roll easily off my tongue when I say them in class."

—*Fabian Hadipriono Tan, Professor of Civil Engineering, The Ohio State University*

NOW YOU TRY!

Create a Metaphor to Help You Learn

Think of a concept you are learning now. Is there another process or idea in a completely different field that somehow seems similar to what you are studying? See if you can come up with a helpful metaphor. (Bonus points if there's a touch of silliness!)

Create Meaningful Groups

Another key to memorization is to create meaningful groups that simplify the material. Let's say you wanted to remember four plants that help ward off vampires—garlic, rose, hawthorn, and mustard. The first letters abbreviate to GRHM, so all you need to do is remember the image of a GRAHAM cracker. (Retrieve your cracker from the kitchen table of your memory palace, dust off the vowels, and you're good to go.)

It's much easier to remember numbers by associating them with memorable events. The year 1965 might be when one of your relatives was born, for example. Or you can associate numbers with a numerical system that you're familiar with. For example, 11.0 seconds is a good running time for the 100-meter dash. Or 75 might be the number of knitting stitches cast onto a needle for the ski hats you like to make. Personally, I like to associate numbers with the feelings of when I was or will be at a given age. The number 18 is an easy one—that's when I went out into the world. By age 104, I will be an old but happy great-granny!

Many disciplines use **memorable sentences** to help students memorize concepts; the first letter of each word in the sentence is also the first letter of each word in a list that needs to be memorized.

Medicine, for example, is laden with memorable mnemonics, among the cleaner of which are "Some Lovers Try Positions that They Can't Handle" (to memorize the names of the carpal bones of the hand) and "Old People from Texas Eat Spiders" (for the cranial bones).

Another example is for the increases-by-ten structure of the decimal system: King Henry died while drinking chocolate milk. This translates to kilo—1,000; hecto—100; deca—10; "while" represents 1; deci—0.1; centi—0.01; milli—0.001.

Time after time, these kinds of memory tricks prove helpful. If you're memorizing something commonly used, see whether someone's come up with a particularly memorable memory trick by searching it out online. Otherwise, try coming up with your own.

BEWARE OF MISTAKING A MEMORY TRICK FOR ACTUAL KNOWLEDGE

"In chemistry we have the phrase *skit ti vicer man feconi kuzin*, which has the cadence of a rap song. It represents the first row of the transition metals on the periodic table (Sc, Ti, V, Cr, Mn, Fe, Co, Ni, Cu, Zn). Then, the rest of the transition metals can be placed on a blank periodic table by other memory tricks. For example, students remember to place Ag (silver) and Au (gold) in the same vertical group as Cu (copper) since copper, silver, and gold are all used to make coins.

"Unfortunately, some students come away thinking that's the *reason* these metals are in the same vertical column—because they are used to make coins. The real reason has to do with similarities in chemical properties and valences.

"This is an example of how students sometimes mistake a memory trick for actual knowledge. Always be wary of confusing *what is truly going on* with the *metaphor* you are using to help your memory."

—William Pietro, Professor of Chemistry, York University, Toronto, Ontario

Create Stories

Notice that the groups mentioned previously often create meaning through story, even though the story might be short. Poor King Henry shouldn't have drunk that chocolate milk! Storytelling in general has long been a vitally important way of understanding and retaining information. Professor Vera Pavri, a historian of science and technology at York University, tells her students not to think of lectures as lectures but as stories where there is a plot, characters, and overall purpose to the discussion. The best lectures in math and science are often framed like thrillers, opening with an intriguing problem that you just *have* to figure out. If your instructor or book doesn't present the material with a question that leaves you wanting to find the answer, see if you can find that question yourself—then set about answering it.[10] And don't forget the value of story as you create memory tricks.

WRITE ON!

"The number one thing I stress when students come to see me is that there is a direct connection between your hand and your brain, and the act of rewriting and organizing your notes is essential to breaking large amounts of information down into smaller digestible chunks. I have many students who prefer to type their notes in a Word document or on slides, and when these students are struggling, the first thing I recommend is to quit typing and start writing. In every case, they perform better on the next section of material."

—Jason Dechant, Ph.D., Course Director, Health Promotion and Development, School of Nursing, University of Pittsburgh